Lecture 3: More binary & overflow

CMPE 120: Computer Organization and Architecture

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Logistics

- HW0 due today!

- Next week's lectures will be async
  - Olivia's OH next week will be rescheduled

Join iClicker

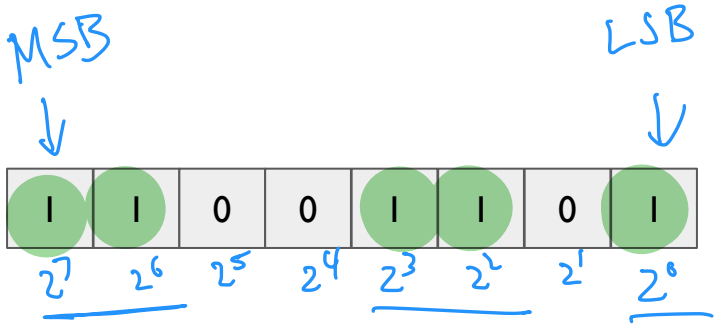
- Join Section 01: https://join.iclicker.com/BXFG

Join iClicker

- Join Section 02: https://join.iclicker.com/RMQT

Review: Binary representation

- What number is this?



A. 200

B. 203

C. 205

D. 206

$\Rightarrow 2^7 + 2^6 + 2^5 + 2^2 + 2^0$

$\Rightarrow 128 + 64 + 8 + 4 + 1$

$\Rightarrow 192 + 13 = 205$

Review: Binary representation

- What number is this?

= 205

Review: Demo

- `uint8_t bin8_to_dec(char bin_arr[])`

How do we add two binary numbers together?

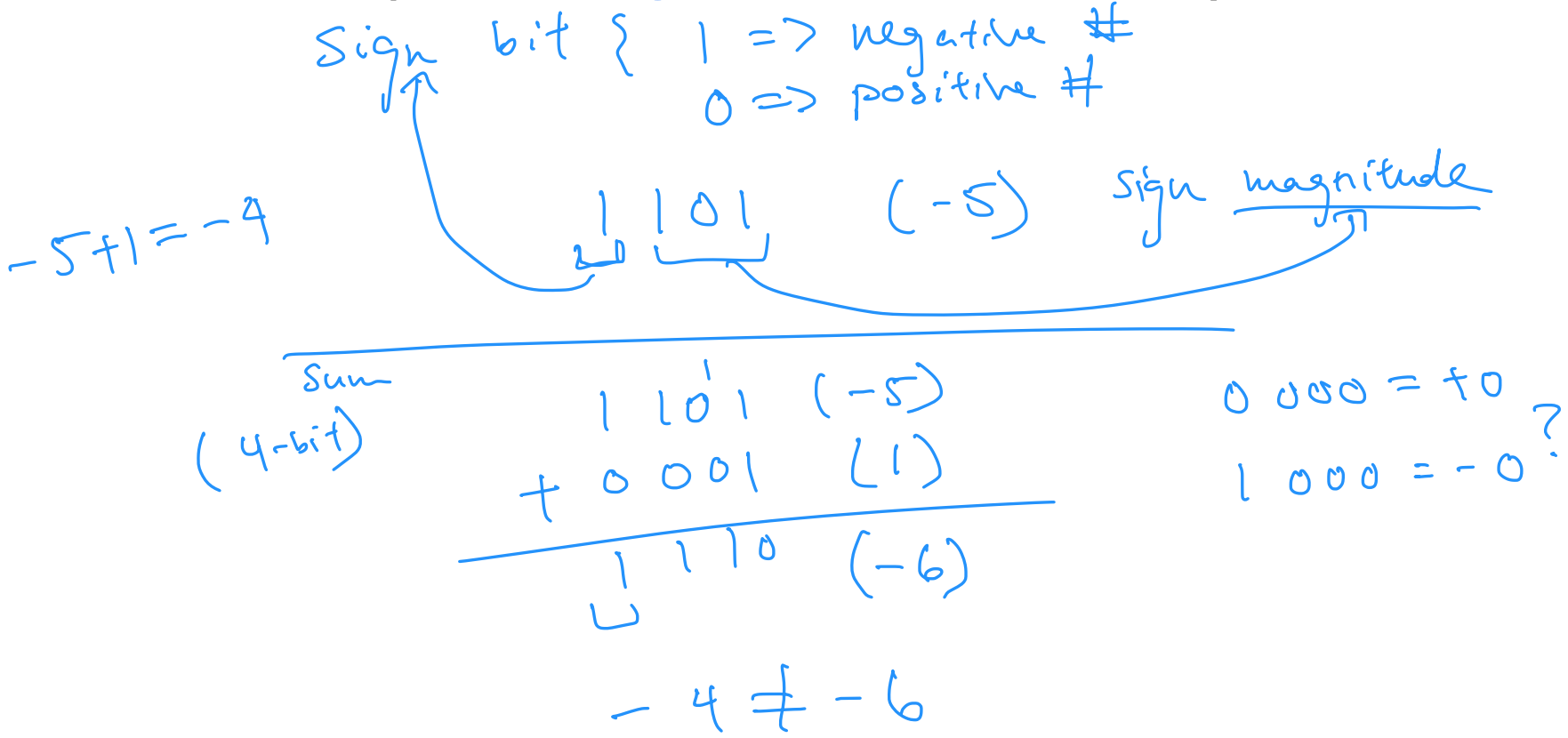
Decimal

```
 15
+ 8
----
 23
```

Binary

```
  1
  01111   (15)
+ 01000   (8)
-------
  10111   (23)
```

How should we represent negative numbers in binary?



How should we represent negative numbers in binary?

- Two's complement
  - signed values = negative & positive #s

-128	64	32	16	8	4	2	1
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	1	1	1	1	1	1

b bits

- Min = $-2^{b-1} \Rightarrow -128$
- Max = $2^{b-1} - 1 \Rightarrow 127$

-64

11000000

$-(2^7) + 2^6 \Rightarrow -128 + 64 = -64$

$-\ldots0100 = 4$

2^2

$1000 = 2^3 = 8$

4-bit

What is the two's complement value?

- 1001

$-(2^3) + 2^0 \Rightarrow -8 + 1 = -7$

$-1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$\Rightarrow -8 + 1 = -7$

- 1111

$-(2^3) + 2^2 + 2^1 + 2^0$

$\Rightarrow -8 + 4 + 2 + 1 = -1$

- 0101

$2^2 + 2^0 = 4 + 1 = 5$

$1010 + 1 = 1011 = -5$?

A. -8 & 4

B. -7 & 5

C. -1 & 5

D. -1 & 4

What is the two's complement value?

- 1001 = -7
- 1111 = -1
- 0101 = 5

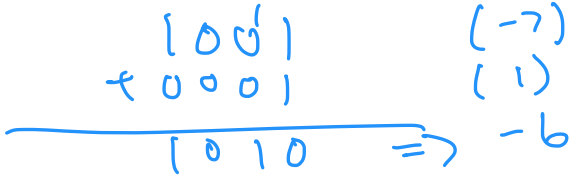
Two's complement

- Signed values in C are represented as two's complement
  - Lets us represent both positive and negative values
  - Example data types: `char`, `int`, `int8_t`

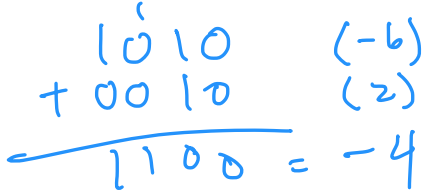
- Unsigned values in C only represent values >= 0
  - Example data types: `unsigned char`, `unsigned int`, `uint8_t`

What is the two's complement sum in binary and decimal?

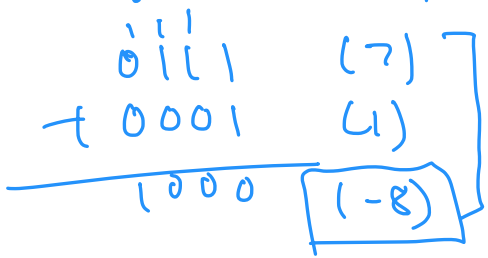
- 1001 + 0001



- 1010 + 0010



- 0111 + 0001



7+1 = 8

A. -6 , 0

B. -4 , -1

C. -5 , 0

D. -4 , 0

E. -4 , -8

What is the two's complement sum in binary and decimal?

- 1001 + 0001 = -7 + 1 = 1010 (-6)
- 1010 + 0010 = -6 + 2 = 1100 (-4)
- 0111 + 0001 = 7 + 1 = 1000 (-8) overflow!

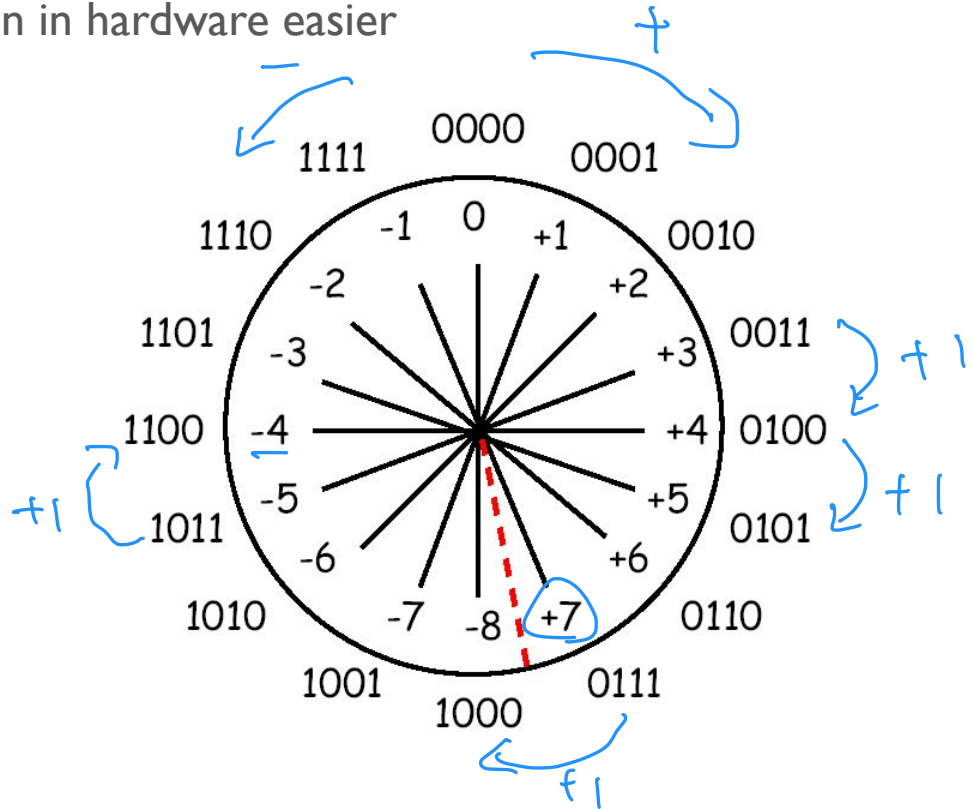
Why do we integer overflow?

- Two's complement makes addition in hardware easier

Ref: https://www.doc.ic.ac.uk/~eedwards/compsys/arithmetic/index.html

Why do we integer overflow?

- Two's complement makes addition in hardware easier
- Fixed number of bits
- Side effect: overflow!



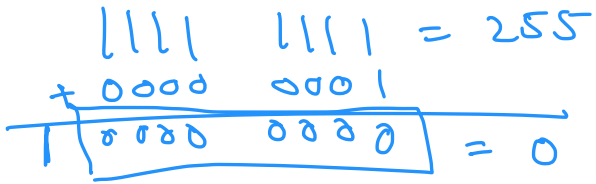
Ref: https://www.doc.ic.ac.uk/~eedwards/compsys/arithmetic/index.html

Consequences of overflow

- Psy's Gangnam Style MV nearly overflowed Youtube's 32-bit view counter
- 1980 Pacman glitched when 8-bit level counter overflows
- Therac-25 radiation therapy machine would overflow, bypassing a safety mechanism, harming 6 patients

Ref: https://diveintosystems.org/book/C4-Binary/overflow.html

Common data types in C

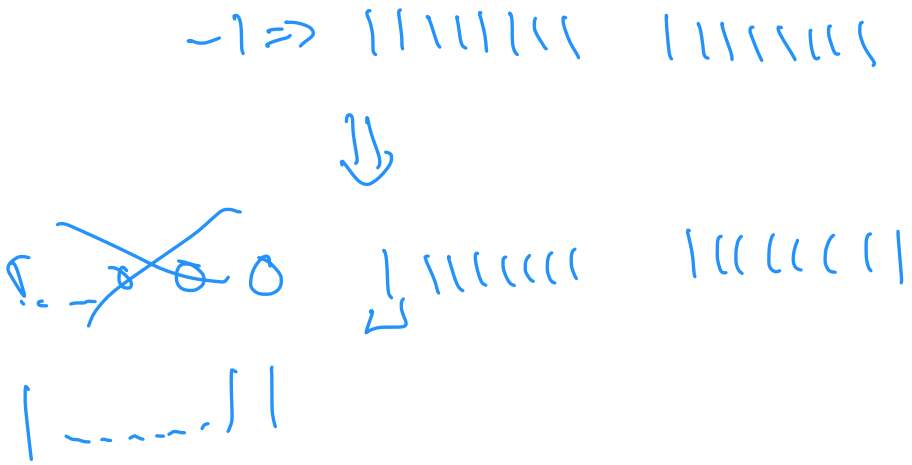


- Integer data types
  - `char = 'A'; // 1 byte signed`
  - `unsigned char = 255; // 1 byte unsigned`
  - `int/int32_t = -5; // 4 bytes signed`
  - `unsigned int/uint32_t = 5; // 4 bytes unsigned`
  - `long long int/int64_t = -10; // 8 bytes signed`
  - `unsigned long long int/uint64_t = 10; // 8 bytes unsigned`

How to copy a small int into larger sized int?

- Need to handle copying a smaller two's complement into a larger one

```
int16_t a_s = -1; // 2 bytes
int a = a_s; // 4 bytes
printf("a = %d a_s = %d\n", a, a_s);
```



A. print same value for both

B. print diffenent values

sign extensions

Practice

Assume a machine where the size of an integer is 1 byte. Assume two's complement to represent signed numbers.

1. What is the max value of an unsigned integer on this machine?

1111 1111

$2^7+2^6+2^5+2^4+2^3+2^2+2^1+2^0 = 255$

2. What is the max value of a signed integer on this machine?

0 111 1111

$0\times(-(2^7))+2^6+2^5+2^4+2^3+2^2+2^1+2^0 = 127$

Practice

Assume a machine where the size of an integer is 1 byte. Assume two's complement to represent signed numbers.

3. What is the minimum value of a signed integer on this machine?

1000 0000

$-(2^7) + 0 + \ldots + 0 = -128$

4. How is the signed integer -1 represented on this machine?

1111 1111

Practice

Assume a machine where the size of an integer is 1 byte. Assume two's complement to represent signed numbers.

5. What is the value of the number `0b10000000` when it is interpreted as

   a. Unsigned integer $2^7 + 0 + \ldots + 0 = 128$

   b. Signed integer $-(2^7) + 0 + \ldots + 0 = -128$