Lecture 3: More binary & overflow

CMPE 120: Computer Organization and Architecture

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Logistics

- HW0 due today!

- Next week's lectures will be async
  - Olivia's OH next week will be rescheduled

Join iClicker

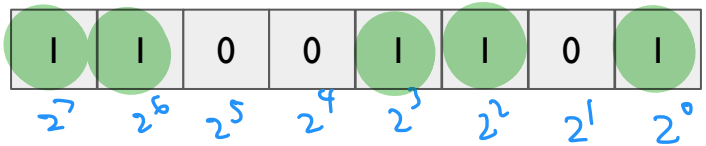
- Join Section 01: https://join.iclicker.com/BXFG

Join iClicker

- Join Section 02: https://join.iclicker.com/RMQT

Review: Binary representation

- What number is this?



A. 203
B. 204
C. 205
D. 206

$2^7 + 2^6 + 2^3 + 2^2 + 2^0$

$128 + 64 + 8 + 4 + 1$

$192 + 13 = 205$ ✓

Review: Binary representation

- What number is this?

= 205

Review: Demo

- `uint8_t bin8_to_dec(char bin_arr[])`

How do we add two binary numbers together?

Decimal

```
  15
+  8
----
  23
```

```
  1
  01111   (15)
+ 01000   (8)
-------
  10111   (23)
```

How should we represent negative numbers in binary?

sign bit — 1 => negative
0 => positive

$-5+1=-4$

1 101
sign magnitude

0 000 = +0
1 000 = -0

$$\begin{array}{r} 1101 \quad (-5) \\ +\ 0001 \quad (1) \\ \hline 1110 \quad (-6) \end{array}$$

$-4 \neq -6$ ∴

How should we represent negative numbers in binary?

- Two's complement
  - signed values — pos. & neg. #s are represented

b = 8

-128	64	32	16	8	4	2	1
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	0	0	0	0	0	0

-64

$-(2^7) + 2^6 + 0 + \ldots + 0 = -128 + 64 = -64$

- Min = -128
- Max = 127

b bits → $min = -2^{b-1}$

→ $max = 2^{b-1} - 1$

4-bit

What is the two's complement value?

- 1001

1001
$-(2^3)\,2^2\,2^1\,2^0$ $\Rightarrow 1\times(-(2^3))+0+0+2^0 = -8+1 = \boxed{-7}$

- 1111 $-(2^3)+2^2+2^1+2^0$
$-8+7 = \boxed{-1}$

- 0101
$0+4+0+1 = \boxed{5}$

A. -8, 4
B. -1, 4
C. -1, 6
D. -1, 5

What is the two's complement value?

- 1001 = -7
- 1111 = -1
- 0101 = 5

Two's complement

- Signed values in C are represented as two's complement
  - Lets us represent both positive and negative values
  - Example data types: `char`, `int`, `int8_t`

- Unsigned values in C only represent values >= 0
  - Example data types: `unsigned char`, `unsigned int`, `uint8_t`

What is the two's complement sum in binary and decimal?

- 1001 + 0001

  -7 + 1 = -6

  1001
  + 0001
  1010 (-6) ✓

- 1010 + 0010 = -4

  1010 (-6)
  + 0010 (2)
  1100 (-4)

- 0111 + 0001

Decimal : 8 = 7 + 1

Binary:

  0111
  + 0001
  1000 (-8)

≠

What is the two's complement sum in binary and decimal?

- 1001 + 0001 = -7 + 1 = 1010 (-6)
- 1010 + 0010 = -6 + 2 = 1100 (-4)
- 0111 + 0001 = 7 + 1 = 1000 (-8) overflow!

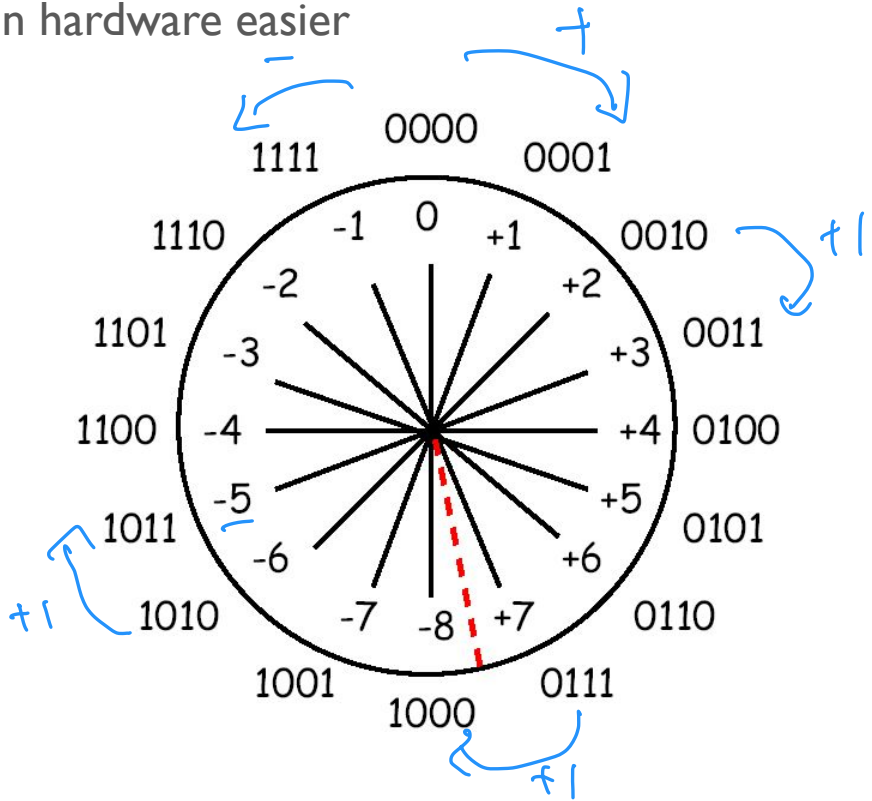
Why do we integer overflow?

- Two's complement makes addition in hardware easier

Ref: https://www.doc.ic.ac.uk/~eedwards/compsys/arithmetic/index.html

Why do we integer overflow?

- Two's complement makes addition in hardware easier
- Fixed number of bits
- Side effect: overflow!



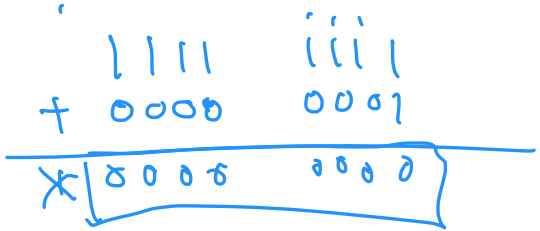
Ref: https://www.doc.ic.ac.uk/~eedwards/compsys/arithmetic/index.html

Consequences of overflow

- Psy's Gangnam Style MV nearly overflowed Youtube's 32-bit view counter
- 1980 Pacman glitched when 8-bit level counter overflows
- Therac-25 radiation therapy machine would overflow, bypassing a safety mechanism, harming 6 patients

Ref: https://diveintosystems.org/book/C4-Binary/overflow.html

Common data types in C

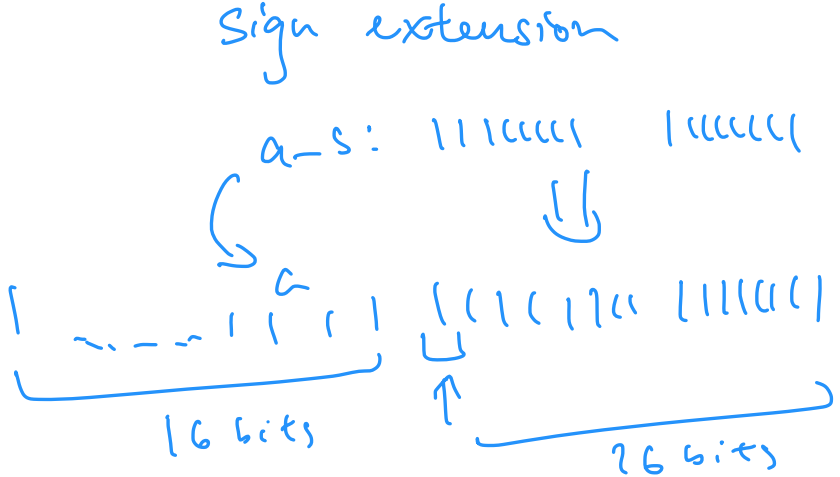


- Integer data types
  - `char = 'A'; // 1 byte signed`
  - `unsigned char = 255; // 1 byte unsigned`
  - `int/int32_t = -5; // 4 bytes signed`
  - `unsigned int/uint32_t = 5; // 4 bytes unsigned`
  - `long long int/int64_t = -10; // 8 bytes signed`
  - `unsigned long long int/uint64_t = 10; // 8 bytes unsigned`

How to copy a small int into larger sized int?

- Need to handle copying a smaller two's complement into a larger one

```
int16_t a_s = -1; // 2 bytes
int a = a_s; // 4 bytes
printf("a = %d a_s = %d\n", a, a_s);
```



A. same value will be printed

B. diff values will be printed

Practice

8 bits

Assume a machine where the size of an integer is 1 byte. Assume two's complement to represent signed numbers.

1. What is the max value of an unsigned integer on this machine?

$b = 8$

$2^b - 1 = 2^8 - 1 = 255$

A. 255
B. 256
C. 128
D. 127

2. What is the max value of a signed integer on this machine?

$\frac{0}{\uparrow}\ \frac{1}{2^6}\ \frac{1}{2^5}\ \frac{1}{2^4}\ \frac{1}{2^3}\ \frac{1}{2^2}\ \frac{1}{2^1}\ \frac{1}{2^0} = 127$

Practice

Assume a machine where the size of an integer is 1 byte. Assume two's complement to represent signed numbers.

3. What is the minimum value of a signed integer on this machine?

1000 0000

$-(2^7) = -128$

→

A. -127

B. -128

C. -255

D. -256

4. How is the signed integer -1 represented on this machine?

1111 1111

Practice

Assume a machine where the size of an integer is 1 byte. Assume two's complement to represent signed numbers.

5. What is the value of the number `0b10000000` when it is interpreted as

a. Unsigned integer

1 0 0 0 0 0 0 0
$2^7 2^6 2^5 2^4$ $2^3 2^2 2^1 2^0$
$= 2^7 =$ **128**

b. Signed integer

1 0 0 0 0 0 0 0
$-(2^7) 2^6 2^5 2^4$ $2^3 2^2 2^1 2^0$
$= -(2^7) =$ **-128**